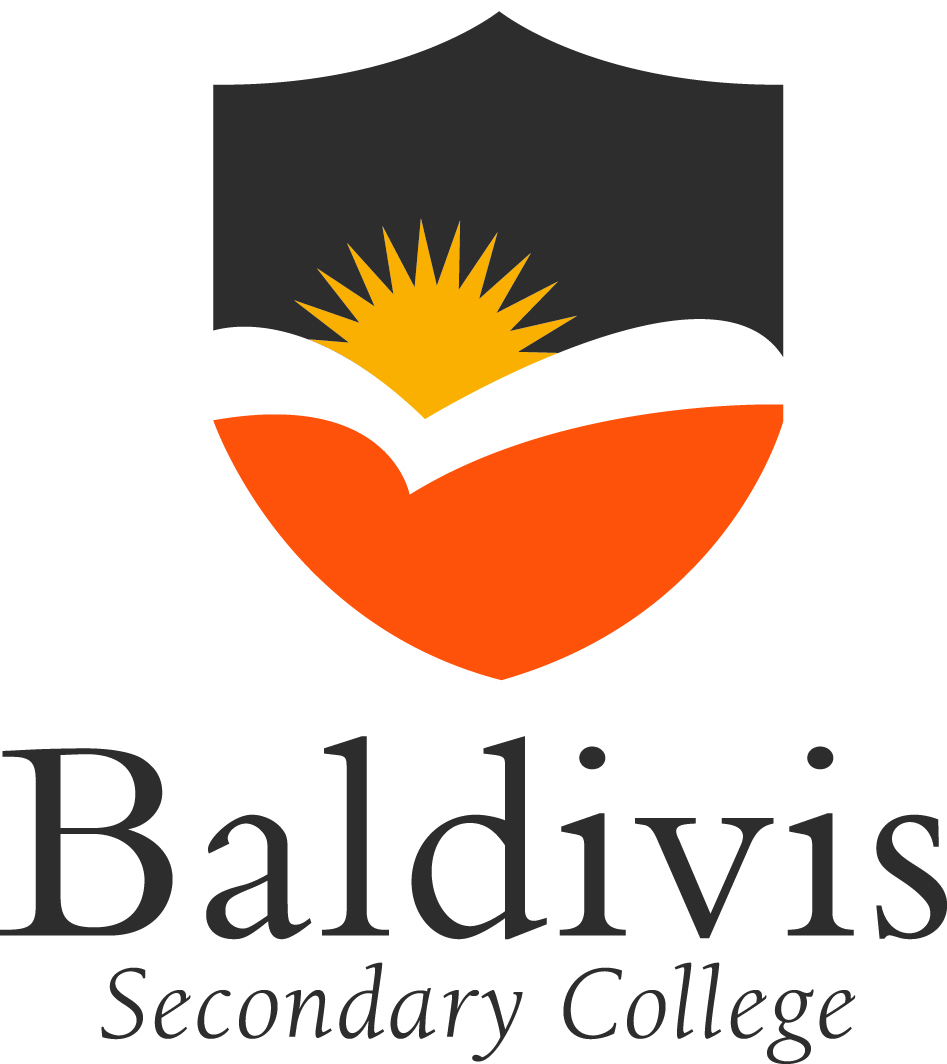
**

**Mathematics Specialist Unit 1**

# Test 6

**Complex Numbers and Proof**

|  |
| --- |
| **Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Total Marks:\_\_\_\_\_\_\_\_\_\_** |
|  |
|  |

**Task type: Response**

**Time allowed for this task:** 55 minutes, in-class, under test conditions

Section One: Calculator-free 35 minutes (34 marks)

Section Two: Calculator-assumed 25minutes ( 26 marks)

**Materials required:** Calculator with CAS capability (to be provided by the student)

**Standard items:** Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

**Special items:**  Drawing instruments, templates, notes on one unfolded sheet of   
A4 paper, and up to three calculators approved for use in the WACE examinations

Formula sheet

**Marks available: 60 marks**

**Task weighting: 8.5%**

**Section One : Calculator Free 34 Marks**

**Time Allowed 35 minutes**

**Question 1**

**[1,1,1,1,2,3=9 marks]**

Let and . Find

✓

✓

✓

✓



**Question 2**

**[3 marks]**

If . Find and .

✓

**Question 3**

**[3 marks]**

Prove that the product of a complex number and its conjugate is a real number.

Let then

(= which is real✓

**Question 4**

**[3 marks]**

The solutions to a quadratic equation are and . Determine the original quadratic equation.

**Question 5**

**[3 marks]**

Prove that is a rational number.

Hence can be expressed as a ratio of two integers and is therefore rational

**Question 6**

**[2,4,3=9 marks]**

Solve the following equations

or

**Question 7**

**[2,2 marks]**

Provide a counter example to disprove the following conjectures.

1. If then.

and .

1. is prime.

If then , which is not prime.

**Section Two : Calculator Assumed 26 Marks**

**Time Allowed 25 minutes Name:\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**Question 8**

**[1,1,1,2=5 marks]**

Show the following numbers as vectors on a single Argand diagram.

|  |  |
| --- | --- |
|  |  |

**Question 9**

**[3 marks]**

Express as the product of two linear factors.

✓

**Question 10**

**[5 marks]**

Use Mathematical induction to prove that  is divisible by 4

If n=1 then , which is divisible by 4

n=k then

n=k+1 then

**Question 11**

**[5 marks]**

Prove by contradiction that is irrational.

Assume that is rational. That is

where with no common factors

is a multiple of 5 and so is a multiple of 5 and

is a multiple of 5 and so is a multiple of 5

This contradicts the assumption that and have no common factors.

Hence the assumption is incorrect and is irrational.

**Question 12**

**[4,4=8 marks]**

Prove the following trigonometric identities